Numerical Modeling of Seepage in Levees with Woody Vegetation

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Modeling Strategy for Seepage

- Select a section of different levee systems in the United States
- Place root zones at various locations on each levee section
- Create 2-D and 3-D finite element models
- Compute variables that influence seepage with and without the presence of root or root zones for both steady-state and transient conditions
  - Exit gradient
  - Pore pressure
  - Total head
  - Velocity
- Analyze results
Computed Variables

- Exit Gradient at Toe of Levee
- Total Head
- Pore Pressure at Bottom of Confining Layer
- Velocity Vectors
Primary Computational Tools

- Groundwater Modeling System (GMS)
- Seep2D
Levee Sites for Modeling

- Albuquerque, New Mexico
  - 1 cross section
- Burlington, Washington
  - 3 cross sections
- Portland, Oregon
  - 1 cross section
- Sacramento, California
  - 1 cross section

Geological and geotechnical properties were provided by levee sponsors, USACE districts and/or existing geotechnical reports.
Portion of Finite Element Mesh

The mesh consisted of 26,614 triangular elements and 13,880 nodes.
Portion of Finite Element Mesh

The mesh consisted of 49,342 triangular elements and 25,395 nodes.
Portland, Oregon

- Silt-clay
- Silty sand
- Sand
- Sandy silt
- Rip-rap

Sand-silt
Portion of Finite Element Mesh

The mesh consisted of 24,939 triangular elements and 12,977 nodes.
Sacramento, California

River elevation

Levee sand

Slurry wall

Clay and sand

Clay and silt

Aquifer sand

Gravel

Silt
Portion of Finite Element Mesh

The mesh consisted of 42,868 triangular elements and 22,139 nodes.
The slurry wall has a height of 20 ft, a top width of 3.5 ft, which narrows to a width of 1.5 ft.
# Original Hydraulic Conductivities

<table>
<thead>
<tr>
<th>Material</th>
<th>$k_H$ (cm/sec)</th>
<th>$k_H$ (ft/day)</th>
<th>$k_V$ (cm/sec)</th>
<th>$k_V$ (ft/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levee sand</td>
<td>$8.00 \times 10^{-3}$</td>
<td>22.7</td>
<td>$2.00 \times 10^{-3}$</td>
<td>5.67</td>
</tr>
<tr>
<td>Clay and silt</td>
<td>$8.00 \times 10^{-4}$</td>
<td>2.27</td>
<td>$2.00 \times 10^{-4}$</td>
<td>0.568</td>
</tr>
<tr>
<td>Clay and sand</td>
<td>$3.00 \times 10^{-5}$</td>
<td>0.085</td>
<td>$1.00 \times 10^{-5}$</td>
<td>0.0283</td>
</tr>
<tr>
<td>Aquifer sand</td>
<td>$8.00 \times 10^{-2}$</td>
<td>226.7</td>
<td>$2.00 \times 10^{-2}$</td>
<td>56.7</td>
</tr>
<tr>
<td>Gravel</td>
<td>$2.00 \times 10^{-2}$</td>
<td>56.7</td>
<td>$2.00 \times 10^{-2}$</td>
<td>56.7</td>
</tr>
<tr>
<td>Silt</td>
<td>$1.00 \times 10^{-4}$</td>
<td>0.283</td>
<td>$1.00 \times 10^{-4}$</td>
<td>0.283</td>
</tr>
<tr>
<td>Slurry wall</td>
<td>$1.00 \times 10^{-6}$</td>
<td>0.00283</td>
<td>$1.00 \times 10^{-6}$</td>
<td>0.00283</td>
</tr>
</tbody>
</table>
Phreatic Surface – Steady-State

Phreatic surface (pore pressure = 0)
Total Head Contours – Steady-State
Hydrograph for Sacramento River
Transient Solution

Initial river level

River level after 3 hr

River level after 5 hr
Root Zone Model Types

- Changes in hydraulic conductivity
- Macropore heterogeneity
- Defect in a levee blanket
  - Assume a simple defect and use Best Practices report
  - Embed an actual root shape into the root zone
Hydraulic Conductivity Root Zone Model

\[ k_{\text{veg}} = \beta k_{\text{orig}} \]
Contours and Vectors for Different $\beta$

- $\beta = 1.0$
- $\beta = 100.0$
- $\beta = 0.01$

Finite Element mesh
## Magnitude of Gradient

<table>
<thead>
<tr>
<th>Points</th>
<th>$\beta = 0.01$</th>
<th>$\beta = 0.1$</th>
<th>$\beta = 1$</th>
<th>$\beta = 10$</th>
<th>$\beta = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.382</td>
<td>0.369</td>
<td>0.284</td>
<td>0.093</td>
<td>0.011</td>
</tr>
<tr>
<td>2</td>
<td>0.263</td>
<td>0.265</td>
<td>0.239</td>
<td>0.087</td>
<td>0.013</td>
</tr>
<tr>
<td>3</td>
<td>0.314</td>
<td>0.305</td>
<td>0.241</td>
<td>0.081</td>
<td>0.011</td>
</tr>
<tr>
<td>4</td>
<td>0.297</td>
<td>0.285</td>
<td>0.217</td>
<td>0.072</td>
<td>0.010</td>
</tr>
<tr>
<td>5</td>
<td>0.282</td>
<td>0.275</td>
<td>0.218</td>
<td>0.073</td>
<td>0.010</td>
</tr>
<tr>
<td>6</td>
<td>0.312</td>
<td>0.296</td>
<td>0.209</td>
<td>0.065</td>
<td>0.009</td>
</tr>
<tr>
<td>7</td>
<td>0.106</td>
<td>0.115</td>
<td>0.168</td>
<td>0.284</td>
<td>0.302</td>
</tr>
<tr>
<td>8</td>
<td>0.134</td>
<td>0.137</td>
<td>0.167</td>
<td>0.248</td>
<td>0.288</td>
</tr>
<tr>
<td>9</td>
<td>0.124</td>
<td>0.128</td>
<td>0.154</td>
<td>0.212</td>
<td>0.237</td>
</tr>
<tr>
<td>10</td>
<td>0.132</td>
<td>0.135</td>
<td>0.154</td>
<td>0.203</td>
<td>0.226</td>
</tr>
</tbody>
</table>
### Pore Pressure

<table>
<thead>
<tr>
<th>Points</th>
<th>$\beta = 0.01$</th>
<th>$\beta = 0.1$</th>
<th>$\beta = 1$</th>
<th>$\beta = 10$</th>
<th>$\beta = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>3</td>
<td>148.1</td>
<td>147.0</td>
<td>139.1</td>
<td>119.4</td>
<td>110.8</td>
</tr>
<tr>
<td>4</td>
<td>193.9</td>
<td>193.7</td>
<td>188.8</td>
<td>165.7</td>
<td>154.5</td>
</tr>
<tr>
<td>5</td>
<td>281.1</td>
<td>278.4</td>
<td>261.4</td>
<td>224.5</td>
<td>208.8</td>
</tr>
<tr>
<td>6</td>
<td>284.7</td>
<td>282.7</td>
<td>268.6</td>
<td>233.9</td>
<td>218.6</td>
</tr>
<tr>
<td>7</td>
<td>512.3</td>
<td>510.0</td>
<td>494.5</td>
<td>457.8</td>
<td>441.6</td>
</tr>
<tr>
<td>8</td>
<td>505.5</td>
<td>503.4</td>
<td>489.2</td>
<td>454.2</td>
<td>438.4</td>
</tr>
<tr>
<td>9</td>
<td>615.8</td>
<td>614.1</td>
<td>602.4</td>
<td>573.4</td>
<td>560.4</td>
</tr>
<tr>
<td>10</td>
<td>609.6</td>
<td>608.1</td>
<td>597.0</td>
<td>568.8</td>
<td>556.0</td>
</tr>
</tbody>
</table>
Observations

§ As $\beta$ is increased (decreased), the magnitude of gradient in the root zone is decreased (increased).

§ As $\beta$ is increased (decreased), the total head contours move away from (toward) the root zone.

§ As $\beta$ is increased (decreased), the flow of water moves toward (away from) the root zone.
Exit Gradients at Toe from Different Root Zone Placements for Three Values of $\beta$

<table>
<thead>
<tr>
<th>Root zone placement</th>
<th>$\beta = 0.01$</th>
<th>$\beta = 1$</th>
<th>$\beta = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Zone near the end of the levee sand on the riverside</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>2 - Zone at the change in slope on the riverside</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>3 - Zone at the river height on the riverside</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>4 - Zone near the top of the landside</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>5 - Zone midway on the steeper landside slope</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>6 - Zone on the toe</td>
<td>0.24</td>
<td>0.33</td>
<td>0.03</td>
</tr>
<tr>
<td>7 - Zone beyond the toe</td>
<td>0.49</td>
<td>0.33</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Observations

§ A root zone placed on or just beyond the toe and at the bottom of a dewatered drainage ditch of a levee changed the exit gradient.

§ Root zones placed at other points along the levee had no impact on the exit gradient at the toe, assuming the absence of long-reaching defects from the roots.
Macropore Heterogeneity Model

Start with 1-inch sized elements in the root zone
Macropore Heterogeneity Model

Randomly vary $\beta_i$ from 0.01 to 100 in all $N$ of the 1-inch sized elements using the random variable, $\eta_i$

$$k_{\text{veg},i} = \beta_i k_{\text{orig}}, \quad \beta_i = 10^{4\eta_i - 2}, \quad 0 \leq \eta_i \leq 1, \quad 1 \leq i \leq N$$
Flow Net with and without Roots

No roots

Roots
2-D Velocity Vectors with Roots
3-D Mesh with Embedded Root Zone
3-D Isosurfaces with Roots
3-D Velocity Vectors with Roots
## Exit Gradient from the Levee Toe to Eight In. from the Toe

<table>
<thead>
<tr>
<th>Distance (in.) from Toe</th>
<th>No root zone</th>
<th>Random set #1</th>
<th>Random set #2</th>
<th>Random set #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (levee toe)</td>
<td>1.57</td>
<td>1.57</td>
<td>1.47</td>
<td>1.69</td>
</tr>
<tr>
<td>1</td>
<td>1.55</td>
<td>2.29</td>
<td>0.58</td>
<td>1.75</td>
</tr>
<tr>
<td>2</td>
<td>1.53</td>
<td>1.02</td>
<td>1.33</td>
<td>1.77</td>
</tr>
<tr>
<td>4</td>
<td>1.61</td>
<td>1.01</td>
<td>1.71</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>1.49</td>
<td>1.47</td>
<td>1.73</td>
<td>0.27</td>
</tr>
<tr>
<td>6</td>
<td>1.47</td>
<td>1.96</td>
<td>2.88</td>
<td>0.54</td>
</tr>
<tr>
<td>7</td>
<td>1.46</td>
<td>1.29</td>
<td>2.88</td>
<td>0.75</td>
</tr>
<tr>
<td>8</td>
<td>1.46</td>
<td>0.38</td>
<td>1.92</td>
<td>0.88</td>
</tr>
</tbody>
</table>
Observations

β Preferred paths can form with much larger velocities than where there is no root.
β 3-D variations are not as much as 2-D ones.
Defect from Root System

Defect

May lead to internal erosion
Simple Defect Model

1-inch defect
Best Practices Toolkit Results

Best Practices Toolkit* assumes the pore pressure underneath the confining layer at the toe is unchanged when there is a defect.

<table>
<thead>
<tr>
<th>Blanket thickness (T) (ft)</th>
<th>30 (original)</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slurry wall</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Average vertical seepage gradient</td>
<td>0.29</td>
<td>1.37</td>
<td>1.37</td>
</tr>
<tr>
<td>Factor of safety (FS) for exit gradient</td>
<td>3.10</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>Probability of a vertical seepage exit</td>
<td>Negligible</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Horizontal gradient</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Probability of Initiation of erosion</td>
<td>Negligible</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Reason for result</td>
<td>T &gt; 25 ft</td>
<td>FS &lt; 1</td>
<td>FS &lt; 1</td>
</tr>
</tbody>
</table>

Comparison of Pore Pressure at the Bottom of the Blanket Below the Toe for a 5-ft Blanket with a 1-inch Defect

<table>
<thead>
<tr>
<th>Defect</th>
<th>Dimension</th>
<th>Pore pressure (lb/ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>2-D</td>
<td>742.4</td>
</tr>
<tr>
<td>Yes</td>
<td>2-D</td>
<td>734.4</td>
</tr>
<tr>
<td>No</td>
<td>3-D</td>
<td>743.6</td>
</tr>
<tr>
<td>Yes</td>
<td>3-D</td>
<td>743.4</td>
</tr>
</tbody>
</table>
Embedded Roots in Root Zone
Contours and Velocity Vectors
Without Root
Contours and Velocity Vectors With Root
Observations

- Best Practices results show that the reliability of a vertical seepage exit and initiation of erosion from a defect at the toe that penetrates the foundation is negligible for the original 30-ft blanket and 1.0 for the 5-ft blanket where 1.0 implies certain to happen.

- A 1-inch defect penetrating the blanket at the toe does not appreciably lower the pore pressure at the bottom of the blanket.
  - Applies to both the simple and embedded real-world defect
  - More apparent for the 3-D results than the 2-D results

- A root zone can shorten the distance between total head contours creating larger exit gradients.
High Performance Computing

- 3-D macropore heterogeneous finite element model had approximately 3,000,000 nodes and 6,000,000 prism elements
- Implicit solution of Richards equation for unsaturated flow requires solution of a linear system of equations at each nonlinear time step
- Requires parallel computing
- Uses 128 cores on the Cray XE6
Richards’ Equation

\[ \frac{\partial}{\partial x} \left( k_r k_H \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_r k_V \frac{\partial \phi}{\partial y} \right) = \frac{\partial \theta}{\partial t} \]

\[ \phi \text{ – total head} \]
\[ \theta \text{ – moisture content} \]
\[ x \text{ – x coordinate} \]
\[ y \text{ – y coordinate} \]
\[ t \text{ – time} \]
\[ k_H \text{ – saturated hydraulic conductivity in the horizontal direction} \]
\[ k_V \text{ – saturated hydraulic conductivity in the vertical direction} \]
\[ k_r \text{ – relative hydraulic conductivity} \]
Computational Challenges

- Hydraulic conductivity \((k)\) in the saturated flow region for different soils (such as sand and clay) can vary several orders of magnitude.
- Horizontal \((k)\) is often four times or more greater than vertical \((k)\).
- For flow in the unsaturated zone, \((k)\) can vary several more orders of magnitude.
- The heterogeneous root zone can have adjacent elements in which the original \((k)\) can differ by as much as four additional orders of magnitude.
- All the above points combine to create ill-conditioned linear systems of equations that must be solved at each nonlinear iteration.
Testing Parallel Linear Iterative Preconditioners and Solvers

- **Preconditioners**
  - Jacobi
  - Block Jacobi
  - Additive Schwarz Method (ASM)
  - Boomer Algebraic Multi-Grid (AMG)

- **Solvers**
  - Conjugate Gradient (CG)
  - Conjugate Residual (CR)
  - Bi-CG Stabilized (BICGS)
  - Generalized Minimal Residual (GMRES)

- Use the Portable, Extensible Toolkit for Scientific Computation (PETSc) library
Solver/Preconditioner Running Time

“X” indicates solver failure
Observations

Regardless of the presence of a root zone:

► The best preconditioner and solver combination is the Boomer AMG preconditioner with the BICGS solver.
► The Boomer AMG preconditioner also performed well with the CG and GMRES solvers.
► The solver and preconditioner combinations of CG/ASM, CR/ASM, and GMRES/Jacobi did not reach a solution.
► Both the BICGS and GMRES solvers required more time to run to reach a solution than either the CG or CR solvers when not combined with the Boomer AMG preconditioner.

The heterogeneity of the root zone produced a level of complexity beyond the capabilities of the following combinations to reach a solution: CG/Jacobi, CG/Block Jacobi, CG/ASM, CR/Jacobi, CR/Block Jacobi, CR/Jacobi, CR/ASM, CR/Boomer AMG, and GMRES/Jacobi.
Overall Conclusions

- The real world is significantly more complicated that the idealizations represented by the models in this presentation.
- Because of the variability in soil properties and root systems, general conclusions are not applicable to all levee systems.
- However, the modeling results can be used as one of several tools to better understand the effect of woody vegetation on levees.
Questions